# 8.2 Proving Trig. Identities

It is often necessary to determine whether two expressions are equivalent to each other. We can use the approaches from the previous section to verify whether equations are identities.

## Multiplying and Factoring Polynomials Involving Trigonometric Functions

We must often multiply binomials or factor trinomials involving trigonometric functions when we verify an identity.

**Examples:**

Multiply $(1+\tan x)(1-\tan x)$

Multiply $(2\sin x+1)^2$

Factor $\sec^2 x-\tan^2 x$

Factor $\sin^2 x+\sin x-2$

## A General Strategy for Verifying Identities

1. Work on the more complicated side first.
2. Rewrite the side you are working with in terms of sines and cosines only.
3. Multiply the numerator and denominator of one rational expression by either the numerator or denominator of the other.
4. Write a single rational expression as a sum of two rational expressions.
5. Combine a sum of two rational expressions into a single rational expression.
6. If both sides simplify to a third expression, then the equation is an identity.

**Examples:**

Verify that $1+\sec x\sin x\tan x=\sec^2 x$ is an identity.

Prove that $\dfrac{\cos\alpha}{1-\sin\alpha}=\dfrac{1+\sin\alpha}{\cos\alpha}$ is an identity.

Prove that $\dfrac{\csc x - \sin x}{\sin x} = \cot^2 x$ is an identity.

Prove that $-2\cot^2 x = \dfrac{1}{1-\sec x} + \dfrac{1}{1+\sec x}$ is an identity.

Prove that $\dfrac{1-\sin^2 t}{1-\csc(-t)} = \dfrac{1+\sin(-t)}{\csc t}$ is an identity.

Show that $\dfrac{1-\cos^2(-t)}{\sin(-t)} = \tan(-t)\cos(-t)$ is an identity.

**A General Strategy for Solving Trigonometric Equations**

1. Know the solutions to $\sin x = 0$, $\cos x = 0$, and $\tan x = 0$.
2. Solve an equation involving multiple angles as if the equation had a single variable.
3. Simplify complicated equations by using identities. If possible, try to get an equation involving only one trigonometric function.
4. If possible, factor to get different trigonometric functions into separate factors.
5. For equations of the quadratic type, solve by factoring or by the quadratic formula.
6. Square each side of the equation, if necessary, so that identities involving squares can be applied. (Remember that this sometimes leads to extraneous solutions—check your answers.)

**Examples:** Find all real number solutions of the following equations.

a) $\sin x \tan x + \sin x = 0$

b) $\sin(2x) = \cos x$

**Examples:** Find all angles in $[0°, 360°)$ that satisfy the following equations.

a) $6\cos^2\left(\frac{x}{2}\right) - 7\cos\left(\frac{x}{2}\right) + 2 = 0$

b) $\cos\alpha - \sin^2\alpha = 0$